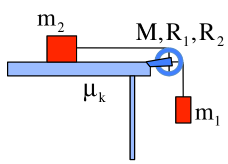
Problem 10.49

The block system is frictional and the pulley is frictionless and massive and can be approximated as a cylinder.



a.) How fast will the blocks be moving after having traveled .700 meters beyond the point where there speeds were .820 m/s?

This is a standard *conservation of energy* problem, with the exception that there is rotational kinetic energy to contend with. I prefer to write out the generic expression, then fill in the blanks. Unfortunately, because we have three objects with both initial and final kinetic energies, plus friction, there isn't room to do it all on one a page (too broad). As such, I am going to do pieces and a lot of manipulation, then put it all together. To start with:

$$I_{cylindricalPully} = \frac{1}{2}M\left[R_1^{\,2} + R_2^{\,2}\right]$$
$$= \frac{1}{2}(.350 \text{ kg})\left[(.0200 \text{ m})^2 + (.0300 \text{ m})^2\right]$$
$$= 2.28\text{x}10^{-4} \text{ kg} \bullet \text{m}^2$$

In algebraic form in all it's glory, the Conservation of Energy reads as:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2 \;(\,\sum U_2 = 0)$$

$$\Rightarrow \left[\left(\frac{1}{2}m_1 v_1^{\,2}\right) + \left(\frac{1}{2}m_2 v_1^{\,2}\right) + \left(\frac{1}{2}I_{pully}\left(\frac{v_1}{R_2}\right)^2\right)\right] + m_1 gh + (-fd) \quad (\tfrac{v_1}{R_2} = \omega_1)$$

$$= \left[\left(\frac{1}{2}m_1 v_2^{\,2}\right) + \left(\frac{1}{2}m_2 v_2^{\,2}\right) + \left(\frac{1}{2}I_{pully}\left(\frac{v_2}{R_2}\right)^2\right)\right]$$

Under normal conditions, we would use N.S.L. to determine the *normal force* so we could determine the *frictional force*. We can see from experience, though, that

$$N = m_2 g \text{ and } W_{friction} = \vec{f} \bullet \vec{d} = (\mu_k N) d \cos 180^\circ = -\mu_k (m_2 g) d$$

Rearranging, we can write our c. of e. expression as:

$$\left[\frac{1}{2}\left(m_1 + m_2 + \frac{I_{pully}}{R_2^{\,2}}\right)v_1^{\,2}\right] + m_1 gh - (\mu_k m_2 g)d = \left[\frac{1}{2}\left(m_1 + m_2 + \frac{I_{pully}}{R_2^{\,2}}\right)v_2^{\,2}\right]$$

Solving for v_2, we get:

$$v_2 = \left[\frac{\left[\frac{1}{2}\left(m_1 + m_2 + \frac{I_{pully}}{R_2^{\,2}}\right)v_1^{\,2}\right] + m_1 gd - (\mu_k m_2 g)d}{\left[\frac{1}{2}\left(m_1 + m_2 + \frac{I_{pully}}{R_2^{\,2}}\right)\right]}\right]^{1/2}$$

$$= \left[v_1^{\,2} + \frac{m_1 gd - (\mu_k m_2 g)d}{\left[\frac{1}{2}\left(m_1 + m_2 + \frac{I_{pully}}{R_2^{\,2}}\right)\right]}\right]^{1/2} = \left[v_1^{\,2} + \frac{gd\left(m_1 - (\mu_k m_2)\right)}{\left[\frac{1}{2}\left(m_1 + m_2 + \frac{I_{pully}}{R_2^{\,2}}\right)\right]}\right]^{1/2}$$

$$= \left[(.820 \text{ m/s})^2 + \frac{(9.80 \text{ m/s}^2)(.700 \text{ m})\left[(.420 \text{ kg}) - (.250)(.850 \text{ kg})\right]}{\left[\frac{1}{2}\left((.420 \text{ kg}) + (.850 \text{ kg}) + \frac{(2.28\text{x}10^{-4} \text{ kg} \bullet \text{m}^2)}{(.0300 \text{ m})^2}\right)\right]}\right]^{1/2}$$

$$= 1.59 \text{ m/s}$$

b.) What is the pulley's angular speed at that point.

This is fairly standard for these kinds of problems (it also goes for N.S.L. problems with rotational and translational motion combined). The first part is sheer hell, whereas the second part is really simple. We know that the edge of the pulley is moving with the same *translational speed* as is the string (otherwise, the string would be slipping over the pulley).

$$v_2 = R_2 \omega_2$$
$$\Rightarrow \omega_2 = \frac{v_2}{R_2}$$
$$= \frac{(1.59 \text{ m/s})}{(.0300 \text{ m})}$$
$$= 53.0 \text{ rad/s}$$